Millimeter-Wave Beams With Phase Singularities

G. Fergus Brand

***Abstract*—Beams of millimeter-wave radiation carrying phase singularities have been generated using specially configured plane and blazed gratings. The blazed grating converts a plane wave into such a beam with very high efficiency. All of the properties of these beams that have been seen at optical wavelengths, such as the hollow profiles of the beams and the way a beam already carrying a singularity is subsequently diffracted, are shown to appear at millimeter wavelengths.**

***Index Terms*—Diffraction, Gaussian beams, gratings, millimeter-wave devices, millimeter-wave propagation.**

I. Introduction

ONE example of a beam with a phase singularity is the higher order Laguerre–Gaussian mode described by

$$E(r, \phi, z) \propto \left(\frac{\sqrt{2}r}{w}\right)^l L_p^l\left(\frac{2r^2}{w^2}\right) \times \exp\left\{-i\kappa z - i(2p+l+1)\arctan\left(\frac{z}{z_r}\right) - \frac{r^2}{w^2} - \frac{i\kappa r^2 z}{2(z_r^2+z^2)} - il\phi\right\} \quad (1)$$

where w is the radius at which the Gaussian term falls to $1/e$ of its on-axis value, L_p^l is an associated Laguerre polynomial, l and p are azimuthal and radial mode numbers, κ is the wavenumber, and z_r is the Rayleigh distance [1]. If $l \neq 0$, this describes a hollow linearly polarized beam. The singularity lies along the axis of the beam. Around a loop drawn about the axis, the phase changes by $l \times 2\ \pi$. The integer l is known as the topological charge. It follows that because there is an ambiguity of phase on the axis, the amplitude must be zero there.

Such phase singularities have been studied at optical wavelengths [2], [3]. These phenomena have also been referred to as wavefront screw dislocations or optical vortices. Much of the interest stems from the fact that a beam like this, with its $\exp(-il\phi)$ azimuthal dependence, carries orbital angular momentum of $l\hbar$ per photon, and any interaction with matter is inevitably accompanied by a transfer of angular momentum [4]. In contrast, a circularly polarized beam is said to carry spin angular momentum [4].

Not unexpectedly, the same phenomena can be observed at millimeter wavelengths. We have previously reported the generation of such beams using diffraction from the following two specially configured gratings: plane [5] and blazed [6]. The blazed grating does this very efficiently. In this paper, a number of new experiments at millimeter wavelengths are described.

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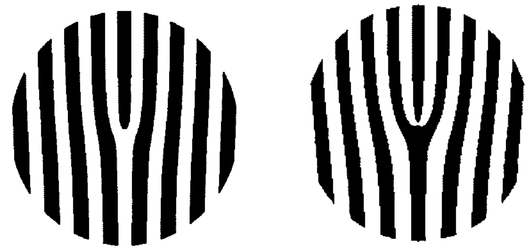

Fig. 1. Plane forked gratings. $D = 7.8$ mm. (a) $p = 1$. (b) $p = 2$.

II. Diffraction of a Plane Wave by a Plane Forked Grating

A. Design of the Gratings

We have used the two varieties of plane forked gratings shown in Fig. 1. Fig. 1(a) shows a grating that can produce a beam of topological charge ± 1. We shall refer to this as a $p = 1$ grating. The boundaries between the transparent and opaque regions are given in polar coordinates (r, θ) by integer values of n where

$$n = \frac{\theta}{\pi} - \frac{2r}{D}\cos\theta \quad (2)$$

while Fig. 1(b) shows a $p = 2$ grating where

$$n = 2\frac{\theta}{\pi} - \frac{2r}{D}\cos\theta + \frac{1}{2}. \quad (3)$$

The integer p refers to the coefficient of the θ/π terms in (2) and (3). D is the grating period at large distances from the center. (If $p = 0$, a strip grating is described.)

For the two plane forked gratings used here, D was 7.8 mm. They were made using printed circuit board so the opaque regions were copper. Only the central part of each grating was used. A circular region 64 mm in diameter was exposed, while the rest of the surface was covered with a microwave absorber.

B. Experiment

The experimental arrangement for viewing diffraction by the plane forked gratings is shown in Fig. 2. The millimeter-wave source was a 105-GHz IMPATT solid-state oscillator with a wavelength λ of 2.86 mm. In contrast to our earlier demonstration [5], two TPX[1] lenses (300-mm focal length,

[1] Trade name for a new polyolefine based on a poly 4 methyl pentene-1.

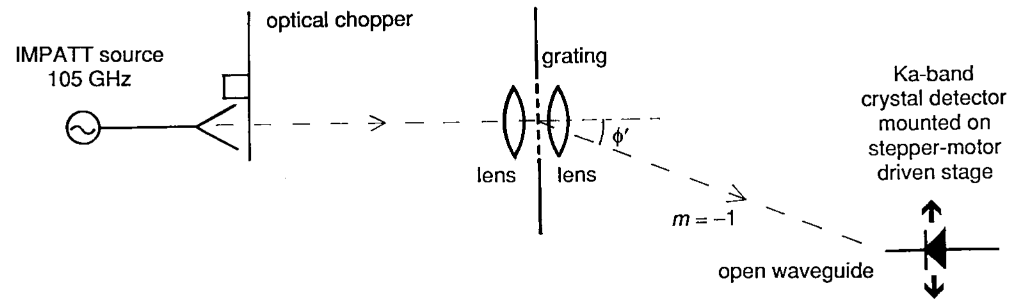

Fig. 2. Experimental arrangement viewed from above. The detector is shown at the position of the first interference maximum.

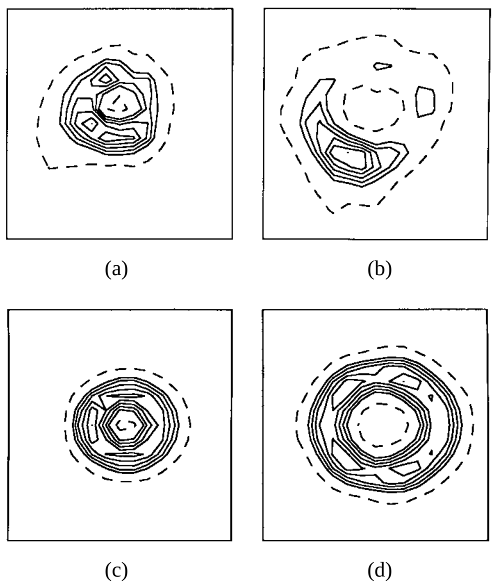

Fig. 3. Intensity contours for the arrangement in Fig. 2, using: (a) the $p = 1$ and (b) $p = 2$ plane forked grating. (c)–(d) Calculated. The area scanned was 80 mm × 80 mm. In all of these figures the contours are for 0.2 (dashed), 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0 of the maximum intensity.

100-mm diameter) were used to ensure Fraunhofer diffraction conditions applied [6]. The apex of the horn is at the focus of the first lens and the diffraction pattern was scanned over an area around the focus of the second lens. The scanning antenna was simply the open end of a *Ka*-band waveguide (7.11 mm × 3.56 mm) mounted with a square-law crystal detector on a stepper-motor-driven x–y translation stage. Throughout these experiments, the polarization of the waves was linear, perpendicular to the plane of the figures.

Most of the diffracted beam passes through the grating in the straight through direction. Only a small fraction is diffracted into the direction of the first interference maximum, the direction shown in Fig. 2, at an angle of $\phi' = 21.5°$. Since signal levels in this direction were quite small, the beam was chopped at the source using an optical chopper and the detected signal was taken to a lock-in amplifier.

The diffraction patterns were obtained by scanning over an area of 80 mm × 80 mm, in steps of 5 mm, centered at $x = 120$ mm, close to the position of the first interference maximum. The patterns obtained are shown in Fig. 3(a) and (b), and the expected patterns are shown in Fig. 3(c) and (d). They show the characteristic hollow beams of phase singularities and the experimental and calculated beam sizes match. These singularities have charges of −1 and −2, respectively. The poorer quality in the second case is because the small amount of diffracted power is spread over a larger area. In all cases, reflections from the lens degrade the experimental patterns. In [5] and [7], the presence of singularities was confirmed by having them interfere with inclined reference beams. The resulting patterns bore some of the forked features of the original gratings.

III. Diffraction of a Plane Wave by a Blazed Forked Grating

A. Blazed Gratings

The Fraunhofer diffraction pattern from a blazed grating can be described as the product of a diffraction term and an interference term [8]. The direction of the principal maximum of the diffraction term is given by the equation for specular reflection from the facets of the grating

$$\phi - \phi' = 2\theta_b \tag{4}$$

and the interference maxima occur where

$$\sin\phi - \sin\phi' = m\,\frac{\lambda}{D}, \qquad m \text{ integer.} \tag{5}$$

If the blaze angle $\theta_b = 10°$ and the period $D = 8.36$ mm, then for an angle of incidence $\phi = 20°$, the $m = 1$ interference maximum will coincide with the principal diffraction maximum and lie in the direction $\phi' = 0°$. Almost all of the power will be diffracted in this direction, in sharp contrast to the small fraction diffracted in the equivalent direction by a plane grating.

B. Design of the Blazed Forked Grating

We have used (2) to design a blazed forked grating [7]. Contours for the crests of the grating (and the troughs immediately below them) are obtained by substituting even integer values of n into (2). The blaze angle was fixed at 10°.

A view of the actual blazed forked grating, milled from an aluminum block, is shown in Fig. 4.

C. Experiment

The diffraction pattern was measured using the arrangement shown in Fig. 5. The beam from the IMPATT source is incident upon the blazed forked grating. Only the central

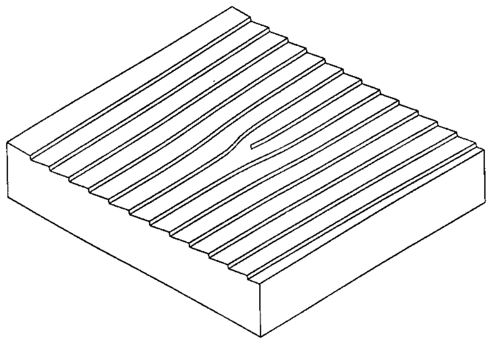

Fig. 4. An isometric view of the $p = 1$ blazed forked grating.

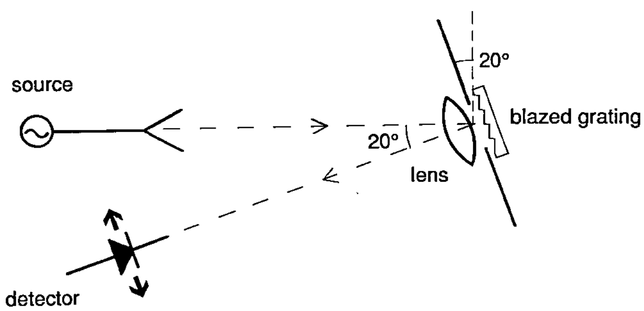

Fig. 5. Experimental arrangement viewed from above.

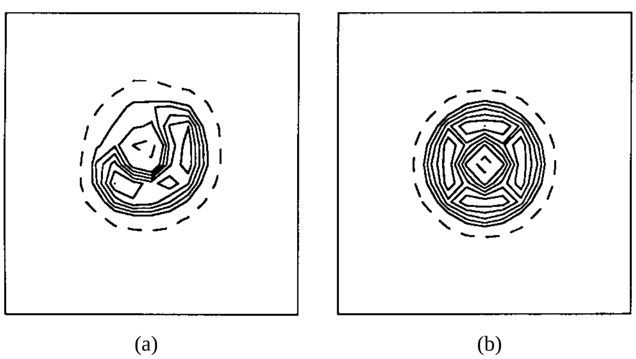

Fig. 6. Intensity contours for the arrangement in Fig. 5 using the blazed forked grating with a circular area exposed. (a) Experimental. (b) Calculated. The area scanned was 80 mm × 80 mm.

64-mm-diameter circle of the blazed grating was used. Again, the TPX lens (300-mm focal length) ensured that Fraunhofer conditions were satisfied. Because the diffraction grating is almost 100% efficient at this wavelength, i.e., almost all of the power is diffracted into the direction of the $m = 1$ interference maximum, the optical chopper and lock-in amplifier were not necessary.

Experimental and calculated patterns are shown in Fig. 6. Again, the hollow nature of the beam is apparent and the beam sizes match. The beam carries a charge of -1.

IV. Diffraction of a Wave Carrying Topological Charge by a Plane Forked Grating

The charge -1 beam produced by the blazed forked grating was used as the source beam for diffraction by a second plane forked grating.

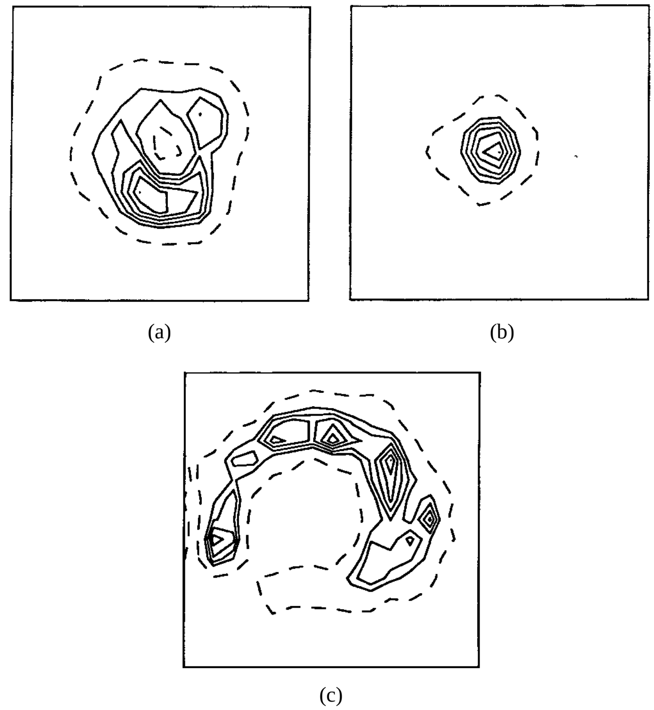

Fig. 7. Intensity contours for the combined arrangement with the blazed forked grating and the $p = 1$ plane forked grating. Experimental beams in the directions of the: (a) $m = 0$, (b) $m = +1$, and (c) $m = -1$ interference maxima. The area scanned was 120 mm × 120 mm.

The results for the case where the second grating was the $p = 1$ plane forked grating are shown in Fig. 7. Fig. 7(a) shows the pattern at the straight through position. The beam is a hollow beam of charge -1, the same as the incident beam. Fig. 7(b) shows the pattern at the position of the $m = +1$ interference maximum. The beam is no longer hollow. There is no phase singularity. The charge -1 of the incident beam and the charge $+1$ of the diffracted beam here combined to yield a charge of 0. Fig. 7(c) shows the pattern at the position of the $m = -1$ maximum. The beam is hollow and larger. This time the charge -1 of the incident beam and the charge -1 of the diffracted beam have combined to yield a charge of -2.

When the second grating is the $p = 2$ plane forked grating, the resulting beams carry charge of -1, $+1$, and -3, respectively.

The "arithmetic" which describes the way the charge of the beam from one grating changes after diffraction by a second grating has been previously seen at optical wavelengths [9].

V. Concluding Remarks

The experiments described here confirm that many of the properties of beams with phase singularities already seen at optical wavelengths also appear at millimeter wavelengths. Such beams are easy to generate with a plane grating, but when high efficiency of conversion is required, the specially configured blazed grating performs extremely well.

Acknowledgment

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REFERENCES

[1] H. Kogelnik and T. Li, "Laser beams and resonators," *Appl. Opt.,* vol. 5, pp. 1550–1567, 1966.
[2] I. V. Basistiy, M. S. Soskin, and M. V. Vasnetsov, "Optical wavefront dislocations and their properties," *Opt. Commun. Opt. Eng., Bellingham,* vol. 119, pp. 604–612, 1995.
[3] N. R. Heckenberg, R. McDuff, C. P. Smith, and A. G. White, "Generation of optical phase singularities by computer-generated holograms," *Opt. Lett.,* vol. 17, pp. 221–223, 1992.
[4] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre–Gaussian laser modes," *Phys. Rev. A, Gen. Phys.*, vol. 45, pp. 8185–8189, 1992.
[5] G. F. Brand, "Generation of millimeter-wave beams with phase singularities," *J. Mod. Opt.,* vol. 44, pp. 1243–1248, 1997.
[6] G. W. Chantry, H. M. Evans, J. W. Fleming, and H. A. Gebbie, "TPX: A new material for optical components in the far infra-red spectral region," *Infrared Phys.*, vol. 9, pp. 31–33, 1969.
[7] G. F. Brand, "The generation of phase singularities at millimeter wavelengths by the use of a blazed grating," *J. Mod. Opt.,* vol. 45, pp. 215–220, 1998.
[8] E. Hecht, *Optics*, 2nd ed. Reading, MA: Addison-Wesley, 1987, pp. 426–427.
[9] V. Yu. Bazhenov, M. S. Soskin, and M. V. Vasnetsov, "Screw dislocations in light wavefronts," *J. Mod. Opt.*, vol. 39, pp. 985–990, 1992.

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